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The Aharonov–Casher effect for spin-1 particles in non-commutative quantum mechanics

S. Dulat[1,3], Kang Li[2,3,a]

[1] School of Physics Science and Technology, Xinjiang University, Urumqi, 830046, P.R. China
[2] Department of Physics, Hangzhou Normal University, Hangzhou, 310036, P.R. China
[3] The Abdus Salam International Center for Theoretical Physics, Trieste, Italy

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Abstract. By using a generalized Bopp's shift formulation, instead of the star product method, we investigate the Aharonov–Casher (AC) effect for a spin-1 neutral particle in non-commutative (NC) quantum mechanics. After solving the Kemmer equations both on a non-commutative space and a non-commutative phase space, we obtain the corrections to the topological phase of the AC effect for a spin-1 neutral particle both on a NC space and a NC phase space.

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1 Introduction

Recently, there has been an increasing interest in the study of physics on a non-commutative space. Apart from non-commutative field theories, there are many papers devoted to the study of various aspects of quantum mechanics on a NC space with the usual (commutative) time coordinate [1–11]. For example, the Aharonov–Bohm phase on a NC space and a NC phase space has been studied in [1–6]. The Aharonov–Casher phase for a spin-1/2 particle on a NC space and a NC phase space has been studied in [7, 8]. Some features of the AC effect for a spin-1 neutral particle on a non-commutative space have been investigated in [9] by using the star product formulation. It is interesting to obtain the corrections to the topological phase of the AC effect for a spin-1 neutral particle both on a NC space and a NC phase space by using the new method in [6].

This article is organized as follows: in Sect. 2, by using the Lagrangian formulation, we discuss the AC effect on a commutative space. In Sect. 3, we study the AC effect on a NC space and give a generalized formula of the AC phase. In Sect. 4, a generalized formula of AC phase on a NC phase space is given. The conclusions are presented in the last section.

2 AC effect for spin-1 particles on a commutative space-time

In this section, following [12], we review briefly the Aharonov–Casher effect of a spin-1 particle on a commutative space time. The Lagrangian for a free spin-1 particle of mass m is

$$L=\bar{\phi}\left(\mathrm{i}\beta^{\nu}\partial_{\nu}-m\right)\phi\,, \tag{1}$$

where the 10×10 matrices β^{ν} are a generalization of the 4×4 Dirac gamma matrices, and they can be chosen as follows [12–15]:

$$\beta^0=\begin{pmatrix}\widehat{O}&\widehat{O}&I&o^{\dagger}\\\widehat{O}&\widehat{O}&\widehat{O}&o^{\dagger}\\I&\widehat{O}&\widehat{O}&o^{\dagger}\\o&o&o&0\end{pmatrix},$$

$$\beta^j=\begin{pmatrix}\widehat{O}&\widehat{O}&\widehat{O}&-iK^{j\dagger}\\\widehat{O}&\widehat{O}&S^j&o^{\dagger}\\\widehat{O}&-S^j&\widehat{O}&o^{\dagger}\\-iK^j&o&o&0\end{pmatrix},$$

with $j=1,2,3$. The elements of the 10×10 matrices β^{ν} are given by the matrices

$$\widehat{O}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix},\quad I=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},$$

$$S^1=\mathrm{i}\begin{pmatrix}0&0&0\\0&0&-1\\0&1&0\end{pmatrix},\quad S^2=\mathrm{i}\begin{pmatrix}0&0&1\\0&0&0\\-1&0&0\end{pmatrix},$$

$$S^3=\mathrm{i}\begin{pmatrix}0&-1&0\\1&0&0\\0&0&0\end{pmatrix},$$

[a] e-mail: kangli@hznu.edu.cn

$$o = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}, \quad K^1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix},$$
$$K^2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}, \quad K^3 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}.$$

The above β matrices satisfy the following relation:

$$\beta_\nu \beta_\lambda \beta_\rho + \beta_\rho \beta_\lambda \beta_\nu = \beta_\nu g_{\lambda\rho} + \beta_\rho g_{\nu\lambda}. \qquad (2)$$

Other algebraic properties of the Kemmer β matrices were given in [13]; the metric tensor is $g_{\lambda\rho} = \mathrm{diag}(1, -1, -1, -1)$. The Kemmer equation of motion is

$$(\mathrm{i}\beta^\nu \partial_\nu - m)\, \phi = 0. \qquad (3)$$

The Lagrangian for a spin-1 neutral particle with a magnetic dipole moment μ_m interacting with the electromagnetic field has the form

$$L = \bar{\phi} \left(\mathrm{i}\beta^\nu \partial_\nu + \frac{1}{2}\mu_m S_{\lambda\rho} F^{\lambda\rho} - m \right) \phi, \qquad (4)$$

where $F^{\lambda\rho}$ is the field strength of the electromagnetic field; $S_{\lambda\rho}$ is the Dirac $\sigma_{\lambda\rho}$-like spin operator, which can be defined as

$$S_{\lambda\rho} = \frac{1}{2}(\beta_\lambda \beta_\rho - \beta_\rho \beta_\lambda). \qquad (5)$$

It follows that in the presence of an electromagnetic field, the Kemmer equation of motion of a spin-1 neutral particle with a magnetic moment μ_m is

$$\left(\mathrm{i}\beta^\nu \partial_\nu + \frac{1}{2}\mu_m S_{\lambda\rho} F^{\lambda\rho} - m \right) \phi = 0. \qquad (6)$$

The aim is to find a solution of the above equation, which can be written in the following form:

$$\phi = \mathrm{e}^{-\mathrm{i}\xi_3 \int^r \mathbf{A}' \mathrm{d}\mathbf{r}} \phi_0, \qquad (7)$$

where ϕ_0 is a solution of (3); the spin-1 pseudo-vector operator ξ_ν in (7) is defined as

$$\xi_\nu = \frac{1}{2}\mathrm{i}\varepsilon_{\nu\lambda\rho\sigma} \beta^\lambda \beta^\rho \beta^\sigma, \qquad (8)$$

where $\varepsilon_{\nu\lambda\rho\sigma}$ is the Levi-Civita symbol in four dimensions. Now we need to find the explicit form of the vector $\mathbf{A}'$ in (7). To do this, first we write (3) for ϕ_0 in terms of ϕ:

$$(\mathrm{i}\beta^\nu \partial_\nu - m)\, \mathrm{e}^{\mathrm{i}\xi_3 \int^r \mathbf{A}' \mathrm{d}\mathbf{r}} \phi = 0. \qquad (9)$$

Then the equivalence of (6) and (9) can be obtained by imposing the following two conditions:

$$\mathrm{e}^{-\mathrm{i}\xi_3 \int^r \mathbf{A}' \mathrm{d}\mathbf{r}} \beta^\nu \mathrm{e}^{\mathrm{i}\xi_3 \int^r \mathbf{A}' \cdot \mathrm{d}\mathbf{r}} = \beta^\nu \qquad (10)$$

and

$$-\beta^\nu \xi_3 A'_\nu \phi = \frac{1}{2}\mu_m S_{\lambda\rho} F^{\lambda\rho} \phi = \mu_m S_{0l} F^{0l} \phi. \qquad (11)$$

By comparing (10) with the Baker–Hausdorff formula,

$$\mathrm{e}^{-\mathrm{i}\lambda\xi_3} \beta^\nu \mathrm{e}^{\mathrm{i}\lambda\xi_3} = \beta^\nu + \wp(-\mathrm{i}\lambda)[\xi_3, \beta^\nu] + \frac{1}{2!}\wp(-\mathrm{i}\lambda)^2 \left[\xi_3, [\xi_3, \beta^\nu]\right] \ldots, \qquad (12)$$

one obtains $[\xi_3, \beta^\nu] = 0$; $\mathfrak{P}$ in (12) stands for the path ordering of the integral in the phase. If $\nu \neq 3$ this commutation relation is automatically satisfied. For $\nu = 3$, by using (2) and (8), one finds that the commutator does not vanish. Therefore in order to fulfill the first condition, the particle is restricted to move in the x–y plane, that is, $p_z = 0$. In particular, $\partial_3 \phi = 0$ and $\hat{A}'_3 = 0$. From the second condition (11), by using (2), (5) and (8), one obtains

$$A'_1 = -2\mu_m E_2, \quad A'_2 = 2\mu_m E_1. \qquad (13)$$

Thus the AC phase for a neutral spin-1 particle moving in a $2+1$ space-time under the influence of a pure electric field produced by a uniformly charged infinitely long filament perpendicular to the plane is

$$\phi_{\mathrm{AC}} = \xi_3 \oint \mathbf{A}' \mathrm{d}\mathbf{r} = 2\mu_m \xi_3 \oint (E_1 \mathrm{d}x_2 - E_2 \mathrm{d}x_1) = 2\mu_m \xi_3 \varepsilon^{lk} \oint E_l \mathrm{d}x_k. \qquad (14)$$

The above equation can also be written as in [12]:

$$\phi_{\mathrm{AC}} = \xi_3 \oint \mathbf{A}' \mathrm{d}\mathbf{r} = \xi_3 \int_S (\nabla \times \mathbf{A}') \mathrm{d}\mathbf{S} = 2\mu_m \xi_3 \int_S (\nabla \mathbf{E}) \mathrm{d}S = 2\mu_m \xi_3 \lambda_e, \qquad (15)$$

where λ_e is the charge density of the filament. This spin-1 AC phase is a purely quantum mechanical effect and has no classical interpretation. One may note that the AC phase for spin-1 particles is exactly the same as in the case of spin-1/2, except that the spin and spinor have changed. The factor of two shows that the phase is twice that accumulated by a spin-1/2 particle with the same magnetic dipole moment coupling constant, in the same electric field.

3 AC effect for spin-1 particles on a non-commutative space

On a NC space the coordinate and momentum operators satisfy the following commutation relations (we take $\hbar = c = 1$ unit):

$$[\hat{x}_i, \hat{x}_j] = \mathrm{i}\Theta_{ij}, \quad [\hat{p}_i, \hat{p}_j] = 0, \quad [\hat{x}_i, \hat{p}_j] = \mathrm{i}\delta_{ij}, \qquad (16)$$

where Θ_{ij} is an element of an antisymmetric matrix; it is related to the energy scale and it represents the non-commutativity of the NC space; $\hat{x}_i$ and $\hat{p}_i$ are the coordinate and momentum operators on a NC space.

By replacing the usual product in (6) with a star product (Moyal–Weyl product), the Kemmer equation for a spin-1 neutral particle with a magnetic dipole moment μ_m, on the NC space, can be written as

$$\left(\mathrm{i}\beta^\nu \partial_\nu + \frac{1}{2}\mu_m S_{\lambda\rho} F^{\lambda\rho} - m \right) * \phi = 0. \qquad (17)$$

The star product between two functions is defined by

$$(f*g)(x) = \mathrm{e}^{\frac{\mathrm{i}}{2}\Theta_{ij}\partial_{x_i}\partial_{x_j}} f(x_i)g(x_j) = f(x)g(x) + \frac{\mathrm{i}}{2}\Theta_{ij}\partial_i f \partial_j g\Big|_{x_i=x_j} + \mathcal{O}(\Theta^2)\,. \tag{18}$$

Here $f(x)$ and $g(x)$ are two arbitrary functions.

On a NC space the star product can be changed into an ordinary product by a Bopp shift, that is, by shifting the coordinates x_ν by

$$x_\nu \to \hat{x}_\nu = x_\nu - \frac{1}{2}\Theta_{\nu\lambda}p^\lambda\,. \tag{19}$$

Now, let us consider the non-commutative Kemmer equation (17). To replace the star product in (17) with an ordinary product, the $F_{\nu\lambda}$ must, up to the first order of the NC parameter Θ, be shifted [6] as

$$F_{\nu\lambda} \to \hat{F}_{\nu\lambda} = F_{\nu\lambda} + \frac{1}{2}\Theta^{\rho\sigma}p_\rho\partial_\sigma F_{\nu\lambda}\,, \tag{20}$$

which is equivalent to a Bopp shift (19). Then the Kemmer equation on a NC space has the form

$$\left(\mathrm{i}\beta^\nu\partial_\nu + \frac{1}{2}\mu_m S_{\lambda\rho}\hat{F}^{\lambda\rho} - m\right)\phi = 0\,. \tag{21}$$

In a similar way as in the commuting space, the solution of the above equation can also be written as

$$\phi = \mathrm{e}^{-\mathrm{i}\xi_3\int^r \hat{\mathbf{A}}'\,\mathrm{d}\mathbf{r}}\phi_0\,. \tag{22}$$

To determine $\hat{\mathbf{A}}'$ we write the free Kemmer equation as

$$(\mathrm{i}\beta^\nu\partial_\nu - m)\,\mathrm{e}^{\mathrm{i}\xi_3\int^r \hat{\mathbf{A}}'\,\mathrm{d}\mathbf{r}}\phi = 0\,. \tag{23}$$

The equivalence of (21) and (23) gives the following two conditions:

$$\mathrm{e}^{-\mathrm{i}\xi_3\int^r \hat{\mathbf{A}}'\,\mathrm{d}\mathbf{r}}\beta^\nu\mathrm{e}^{\mathrm{i}\xi_3\int^r \hat{\mathbf{r}}'\,\mathrm{d}\mathbf{r}} = \beta^\nu \tag{24}$$

and

$$-\beta^\nu\xi_3\hat{A}'_\nu\phi = \frac{1}{2}\mu_m S_{\lambda\rho}\hat{F}^{\lambda\rho}\phi = \mu_m S_{0l}\hat{F}^{0l}\phi\,. \tag{25}$$

By using the Baker–Hausdorff formula, (12), the first condition (24) implies that $[\xi_3,\beta^\nu]=0$. If $\nu\neq 3$, then this commutation relation is automatically satisfied. For $\nu = 3$, by using (2) and (8), one finds that the commutator does not vanish. Therefore in order to fulfill the first condition we restrict ourselves to $2+1$ space-time. In particular, $\partial_3\phi = 0$ and $\hat{A}'_3 = 0$. From the second condition (25), by using (2), (5) and (8), one obtains

$$\begin{aligned}\hat{A}'_1 &= -2\mu_m\hat{F}^{02} = -2\mu_m F^{02} - 2\mu_m\frac{1}{2}\Theta^{ij}p_i\partial_j F^{02}\\ &= -2\mu_m E_2 - \mu_m\theta\varepsilon^{ij}p_i\partial_j E_2\,,\\ \hat{A}'_2 &= 2\mu_m\hat{F}^{01} = 2\mu_m F^{01} + 2\mu_m\frac{1}{2}\Theta^{ij}p_i\partial_j F^{01}\\ &= 2\mu_m E_1 - \mu_m\theta\varepsilon^{ij}p_i\partial_j E_1\,,\end{aligned} \tag{26}$$

with $\Theta^{ij} = \theta\epsilon^{ij}$, $\Theta^{0\mu} = \Theta^{\mu 0} = 0$; $\epsilon^{ij} = -\epsilon^{ji}$ and $\epsilon^{12} = +1$. Thus the AC phase for a neutral spin-1 particle moving in a $2+1$ non-commutative space under the influence of a pure electric field produced by a uniformly charged infinitely long filament perpendicular to the plane is

$$\hat{\phi}_{\mathrm{AC}} = \xi_3\oint \hat{\mathbf{A}}'\,\mathrm{d}\mathbf{r} = 2\mu_m\xi_3\varepsilon^{lk}\oint E_l\,\mathrm{d}x_k + \mu_m\xi_3\theta\varepsilon^{ij}\varepsilon^{lk}\oint p_i\partial_j E_l\,\mathrm{d}x_k\,. \tag{27}$$

In a similar way as in the spin-$\frac{1}{2}$ case [6, 8], the momentum on a NC space for a spin-1 neutral particle can also be written as

$$p_i = mv_i + (\mathbf{E}\times\mu)_i + \mathcal{O}(\theta)\,, \tag{28}$$

where $\mu = 2\mu_m\mathbf{S}$, and $\mathbf{S}$ is the spin operator of spin-1. By inserting (28) into (27), we have

$$\hat{\phi}_{\mathrm{AC}} = \phi_{\mathrm{AC}} + \delta\phi_{\mathrm{NCS}}\,, \tag{29}$$

where ϕ_{AC} is the AC phase in (14) on a commuting space; the additional phase $\delta\phi_{\mathrm{NCS}}$, related to the non-commutativity of space, is given by

$$\delta\phi_{\mathrm{NCS}} = \mu_m\xi_3\theta\varepsilon^{ij}\varepsilon^{lk}\oint[k_i - (\mu\times\mathbf{E})_i]\partial_j E_l\,\mathrm{d}x_k\,, \tag{30}$$

where $k_i = mv_i$ is the wave number; ξ_3 represents the spin degrees of freedom. If the spin of the neutral particle is along the z direction, namely, $\mu = 2\mu_m s_3\hat{\mathbf{k}}$, where $\hat{\mathbf{k}}$ is a unit vector in the z direction, then our results here are the same as the result of [9], where the star product calculation has been used.

4 AC effect for spin-1 particles on a non-commutative phase space

In Sect. 3 we have investigated the AC effect for a neutral spin-1 particle on a NC space, where space–momentum, and space–space are non-commuting, but momentum–momentum are commuting. Bose–Einstein statistics in non-commutative quantum mechanics requires both space–space and momentum–momentum to be non-commuting. The NC space with non-commuting momentum–momentum is called NC phase space. In this section we study the AC phase on a NC phase space. On a NC phase space, the commutation relation in (16) should be replaced by

$$[\hat{p}_i,\hat{p}_j] = \mathrm{i}\bar{\Theta}_{ij}\,, \tag{31}$$

where $\bar{\Theta}$ is the antisymmetric matrix, and its elements represent the non-commutative property of the momenta. Then the Kemmer equation for the AC problem on a NC phase space has the form

$$\left(-\beta^\nu p_\nu + \frac{1}{2}\mu_m S_{\lambda\rho}F^{\lambda\rho} - m\right)*\phi = 0\,. \tag{32}$$

The star product in (32) on a NC phase space can be replaced by the usual product in two steps; first we need to replace x_i and p_i by a generalized Bopp shift as

$$x_\nu \to \hat{x}_\nu = \alpha x_\nu - \frac{1}{2\alpha}\Theta_{\nu\lambda}p^\lambda ,$$
$$p_\nu \to \hat{p}_\nu = \alpha p_\nu + \frac{1}{2\alpha}\bar{\Theta}_{\nu\lambda}x^\lambda , \quad (33)$$

where α is the scaling parameter, and it is related to the non-commutativity of the phase space via $\Theta\bar{\Theta} = 4\alpha^2(\alpha^2 - 1)\cdot \mathbf{I}$, where $\mathbf{I}$ is a unit matrix. Then we also need to rewrite the shift in (20) as

$$F_{\nu\lambda} \to \hat{\mathcal{F}}_{\nu\lambda} = \alpha F_{\nu\lambda} + \frac{1}{2\alpha}\Theta^{\rho\sigma}p_\rho\partial_\sigma F_{\nu\lambda} . \quad (34)$$

Thus the Kemmer equation for the AC problem on a NC phase space has the form

$$\left(-\beta^\nu \hat{p}_\nu + \frac{1}{2}\mu_m S_{\lambda\rho}\hat{\mathcal{F}}^{\lambda\rho} - m\right)\phi = 0 . \quad (35)$$

Since $\alpha \neq 0$, the above equation can be written as

$$\Bigg(-\beta^\nu p_\nu - \frac{1}{2\alpha^2}\beta^\nu\bar{\Theta}_{\nu\lambda}x^\lambda + \frac{1}{2}\mu_m S_{\lambda\rho} \times \left(F^{\lambda\rho} + \frac{1}{2\alpha^2}\Theta^{\sigma\tau}p_\sigma\partial_\tau F^{\lambda\rho}\right) - m'\Bigg)\phi = 0 , \quad (36)$$

where $m' = m/\alpha$. We write the above equation in the following form:

$$(-\beta^\nu p_\nu - m')\, \mathrm{e}^{\frac{\mathrm{i}}{2\alpha^2}\int^r \bar{\Theta}_{\nu\lambda}x^\lambda \,\mathrm{d}x^\nu + \mathrm{i}\xi_3\int^r \hat{\mathcal{A}}'\,\mathrm{d}\mathbf{r}}\phi = 0 . \quad (37)$$

To have equivalence of (36) and (37), we impose the following two conditions:

$$\mathrm{e}^{-\mathrm{i}\xi_3\int^r \hat{\mathcal{A}}'\,\mathrm{d}\mathbf{r}}\beta^\nu \mathrm{e}^{\mathrm{i}\xi_3\int^r \hat{\mathbf{A}}'\,\mathrm{d}\mathbf{r}} = \beta^\nu \quad (38)$$

and

$$-\beta^\nu\xi_3\hat{\mathcal{A}}'_\nu\phi = \frac{1}{2\alpha}\mu_m S_{\lambda\rho}\hat{F}^{\lambda\rho}\phi = \frac{\mu_m}{\alpha}S_{0l}\hat{F}^{0l}\phi . \quad (39)$$

In an analogous way as in NC space, from (38) and (39) one may obtain

$$\begin{aligned}
\hat{\mathcal{A}}'_1 &= -\frac{2\mu_m}{\alpha}\hat{F}^{02} = -2\mu_m F^{02} - 2\mu_m\frac{1}{2\alpha^2}\Theta^{ij}p_i\partial_j F^{02} \\
&= -2\mu_m E_2 - \frac{\mu_m\theta}{\alpha^2}\varepsilon^{ij}p_i\partial_j E_2 , \\
\hat{\mathcal{A}}'_2 &= \frac{2\mu_m}{\alpha}\hat{F}^{01} = 2\mu_m F^{01} + 2\mu_m\frac{1}{2\alpha^2}\Theta^{ij}p_i\partial_j F^{01} \\
&= 2\mu_m E_1 + \frac{\mu_m\theta}{\alpha^2}\theta\varepsilon^{ij}p_i\partial_j E_1 , \\
\hat{\mathcal{A}}'_3 &= 0 .
\end{aligned} \quad (40)$$

Thus the AC phase for a neutral spin-1 particle moving in a $2+1$ non-commutative phase space under the influence of a pure electric field produced by a uniformly charged infinitely long filament perpendicular to the plane is given by

$$\begin{aligned}
\hat{\varphi}_{\mathrm{AC}} &= \frac{1}{2\alpha^2}\oint \bar{\Theta}_{\nu\lambda}x^\lambda\,\mathrm{d}x^\nu + \xi_3\oint \hat{\mathcal{A}}'\,\mathrm{d}\mathbf{r} \\
&= \frac{\theta}{2\alpha^2}\oint \varepsilon^{ij}x_j\,\mathrm{d}x_i + 2\mu_m\xi_3\varepsilon^{lk}\oint E_l\,\mathrm{d}x_k \\
&\quad + \mu_m\xi_3\frac{\theta}{\alpha^2}\varepsilon^{ij}\varepsilon^{lk}\oint p_i\partial_j E_l\,\mathrm{d}x_k .
\end{aligned} \quad (41)$$

By $p_i = k'_i + (\mathbf{E}\times\mu)_i + \mathcal{O}(\theta)$, $k'_i = m'_i v_i$, with $\mu = 2\mu_m\mathbf{S}$, one obtains

$$\hat{\varphi}_{\mathrm{AC}} = \phi_{\mathrm{AC}} + \delta\phi_{\mathrm{NCS}} + \delta\phi_{\mathrm{NCPS}} , \quad (42)$$

where ϕ_{AC} is the AC phase in (14) on a commuting space; $\delta\phi_{\mathrm{NCS}}$ is the space–space non-commuting contribution to the AC phase in (14), and its explicit form is given in (30); the last term $\delta\phi_{\mathrm{NCPS}}$ is the momentum–momentum non-commuting contribution to the AC phase in (14), and it has the form

$$\begin{aligned}
\delta\phi_{\mathrm{NCPS}} &= \frac{\bar{\theta}}{2\alpha^2}\oint \varepsilon^{ij}x_j\,\mathrm{d}x_i + \left(\frac{1}{\alpha^2} - 1\right)\mu_m\xi_3\theta\varepsilon^{ij}\varepsilon^{lk} \\
&\quad \times \oint [k'_i - (\mu\times\mathbf{E})_i]\,\partial_j E_l\,\mathrm{d}x_k ,
\end{aligned} \quad (43)$$

which represents the non-commutativity of the momenta. The first term in (43) comes from the momentum–momentum non-commutativity; the second term is a velocity dependent correction, which does not have the topological properties of the commutative AC effect and could modify the phase shift; the third term is a correction to the vortex; it does not contribute to the line spectrum. In the two dimensional non-commutative plane, $\bar{\Theta}_{ij} = \bar{\theta}\epsilon_{ij}$, and the two NC parameters θ and $\bar{\theta}$ are related by $\bar{\theta} = 4\alpha^2(1-\alpha^2)/\theta$ [11]. When $\alpha = 1$, which leads to $\bar{\theta}_{ij} = 0$, the AC phase on a NC phase space case reduces to the AC phase on a NC space case, i.e. $\delta\phi_{\mathrm{NCPS}} = 0$ and (42) changes into (29).

5 Conclusions

In this paper in order to study the AC effect both on a non-commutative space and on a non-commutative phase space, we use the shift method, instead of the star product formulation. Our shift method is equivalent to the star product method, i.e., the Kemmer equation with star product can be replaced by a Bopp shift together with the shift that we defined in (20) for a NC space and in (34) for a NC phase space. The additional AC phase in (29) on a NC space is the same as the result of [9], where the star product method has been used. Furthermore, by considering the momentum–momentum non-commutativity we obtained the NC phase space corrections to the topological phase of the AC effect for a spin-1 neutral particle. We note that the corrections (30) and (43) to the topological phase (14) or (15) of the AC effect for a spin-1 neutral particle both on a NC space and a NC phase space can be obtained from spin-1/2 corrections through the replacement

$\frac{1}{2}\gamma^0\sigma^{12} \longrightarrow \xi_3$. One may conclude that, apart from the spin operators, the AC phase for a higher spin neutral particle is the same as in the cases of spin-1/2 and spin-1 in non-commutative quantum mechanics.

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